XVI. *On the Direction and Velocity of the Motion of the Sun, and Solar System.* *By* William Herschel, *LL. D. F. R. S.*

Read May 16, 1805.

Our attention has lately been directed again to the construction of the heavens, on which I have already delivered several detached papers. The changes which have taken place in the relative position of double stars, have ascertained motions in many of them, which are probably of the same nature with those that have hitherto been called proper motions. It is well known that many of the principal stars have been found to have changed their situation, and we have lately had a most valuable acquisition in Dr. Maskelyne's Table of proper motions of six and thirty of them. If this Table affords us a proof of the motion of the stars of the first brightness, such as are probably in our immediate neighbourhood, the changes of the position of minute double stars that I have ascertained, many of which can only be seen by the best telescopes, likewise prove that motions are equally carried on in the remotest parts of space which hitherto we have been able to penetrate.

The proper motions of the stars have long engaged the attention of astronomers, and in the year 1783, I deduced from them, with a high degree of probability, a motion of the sun and solar system towards λ Herculis. The reasons which were then pointed out for introducing a solar motion, will now be much strengthened by additional considerations; and the

above mentioned Table of well ascertained proper motions will also enable us to enter rigorously into the necessary calculations for ascertaining its direction, and discovering its velocity. When these points are established, we shall be prepared to draw some consequences from them that will account for many phenomena which otherwise cannot be explained.

The scope of this Paper, wherein it is intended to assign not only the direction, but also the velocity of the solar motion, embraces an extensive field of observation and calculation; but as to give the whole of it would exceed the compass of the present sheets, I shall reserve the velocity of the solar motion for an early future opportunity, and proceed now to a disquisition of the first part of my subject, which is the direction of the motion of the sun and solar system.

Reasons for admitting a solar Motion.

It may appear singular that, after having already long ago pointed out a solar motion, and even fixed upon a star towards which I supposed it to be directed, I should again think it necessary to show that we have many substantial reasons for admitting such a motion at all. What has induced me to enter into this inquiry is, that some of the consequences hereafter to be drawn from a solar motion when established, seem to condradict the very intention for which it is to be introduced. The chief object in view, when a solar motion was proposed to be deduced from observations of the proper motions of stars, was to take away many of these motions by investing the sun with a contrary one. But the solar motion, when its existence has been proved, will reveal so many concealed real motions, that we shall have a greater sum of them than it would be necessary

to admit, if the sun were at rest; and, to remove this objection, the necessity for admitting its motion ought to be well established.

Theoretical Considerations.

A view of the motion of the moons, or secondary planets, round their primary ones, and of these again round the sun, may suggest the idea of an additional motion of the latter round some other unknown center; and those who like to indulge in fanciful reviews of the heavens, might easily build a system upon hypotheses not altogether without some plausibility in their favour. Accordingly we find that Mr. LAMBERT, in a work which is full of the most fantastic imaginations, has framed a system wherein the sun is supposed to move about the nebula in Orion.* But, setting aside the extravagant idea of making this luminous spot a center of motion, it must certainly be admitted that the solar motion itself is at least a very possible event.

I have already mentioned, in a note to my former Paper,† that the possibility of a solar motion has also been shown from theoretical principles by the late Dr. WILSON of Glasgow; and its probability afterwards, from reasons of the same nature, by Mr. DE LA LANDE. The rotatory motion of the sun, from which he concludes a displacing of the solar center, must certainly be allowed to indicate a motion of translation in space; for though it may be possible, it does not appear probable, that any mechanical impression should have given the former, without occasioning the latter. But, as we are intirely unacquainted with the cause of the rotatory motion, the solar

* See *Système du Monde de* Mr. LAMBERT, page 152, and 158.

† See Phil. Trans. for the year 1783, page 283.

translation in space from theoretical reasons, can only be admitted as a very plausible hypothesis.

It would be worth while for those who have fixed instruments, to strengthen this argument by observing the stars which are known to change their magnitudes periodically. For as we have great reason to ascribe these regular changes to a rotatory motion of the stars,* a real motion in space may be expected to attend it; and the number of these stars is so considerable that their concurring testimony would be very desirable.

Perhaps Algol, which according to these ideas must have a very quick rotatory motion, may be found to have also a considerable progressive one; and if that should be ascertained, the position of the axis of the rotation of this star will be in a great measure thereby discovered.

An argument from the real motion to a rotatory one is nearly of equal validity, and therefore all the stars that have a motion in space may be surmised to have also a rotation on their axes.

Symptoms of parallactic Motions.

But, setting aside theoretical arguments, I shall now proceed to such as may be drawn from observation; and, as all parallactic motions are evident indications that the observer of them is not at rest, it will be necessary to explain three sorts of motions, of which the parallactic is one; they will often engage our attention in the following discussion.

Let the sun be supposed to move towards a certain part of the heavens, and since the whole solar system will have the

* See Phil. Trans. for the year 1795, page 68.

same motion, the stars must appear to an inhabitant of the earth to move in an opposite direction. In the triangle *s p a*, Plate VII. Fig. 1, let *s p* represent the parallactic motion of a star; then, if this star is one that has no real motion, *sp* will also be its apparent motion; but if the star in the same time, that by its parallactic motion it would have gone from *s* to *p*, should have a real motion which would have carried it from *s* to *r*, then will it be seen to move along the diagonal *s a*, of the parallelogram *s r p a;* and *p a*, which is parallel and equal to *s r*, will represent its real motion. Therefore, in the above mentioned triangle *s p a*, which I suppose to be formed in the concave part of the heavens by three arches of great circles, the eye of the observer being in the center, the three sides will represent, or stand for, the three motions I have named: *sp* the parallactic, *pa* the real, and *sa* the apparent motion of the star. The situation and length of these arches, in seconds of a degree, will express, or rather represent, not only the direction but also the quantity of each motion, such as it must appear to an eye in the above mentioned central situation. And calling the solar motion S, the distance of the star from the sun *d*, and the sine of the star's distance from the point towards which the sun is moving φ, the parallactic motion, when these are given, will be had by the expression $\frac{\varphi \cdot S}{r \cdot d} = sp$. This theorem, and its corollaries, of which frequent use will be made hereafter, it will not be necessary here to demonstrate.

When I call the arch *pa* the real motion, it should be understood that I only mean its representative; for it must be evident that the absolute motion of a star in space, as well as its intrinsic velocity, will still remain unknown, because the inclination of that motion on which also its real velocity will

depend, admits of the greatest variety of directions. We are only acquainted with the plane in which the motion must be performed, and with the length of the arch in seconds by which that motion may be measured. We may add that the chords of the arches representing the three motions are the smallest velocities of these motions that can be admitted; for in every other direction but at right angles to the line of sight, the actual space over which the star will move must be greater than the arch or chord by which its motion is represented.

Now, since a motion of the sun will occasion parallactic motions of the stars, it follows that these again must indicate a solar motion; but in order to ascertain whether parallactic motions exist, we ought to examine those stars which are most liable to be visibly affected by solar motion. This requisite points out the brightest stars as the most proper for our purpose; for any star may have a great real motion, but in order to have a great parallactic one, it must be in the neighbourhood of the sun. And as we can only judge of the distance of the stars by their splendour we ought to choose the brightest, on account of a probability that, being nearer than faint ones, they may be more within the reach of parallax, and thus better qualified to show its effects.

We are also to look out for a criterion whereby parallactic may be distinguished from real motions; and this we find in their directions. For if a solar motion exists, all parallactic motions will tend to a point in opposition to the direction of that motion; whereas real motions will be dispersed indiscriminately to all parts of space.

With these distinctions in view, we may examine the proper motions of the principal stars; for these, if the sun is not at

rest, must either be intirely parallactic, or at least composed of real and parallactic motions; in the latter case they will fall under the denomination of one of the three motions we have defined, namely *sa*, the apparent motion of the star.

In consequence of this principle I have delineated the meeting of the arches arising from a calculation of the proper motions of the 36 stars in Dr. MASKELYNE's catalogue, on a celestial globe; and, as all great circles of a sphere intersect each other in two opposite points, it will be necessary to distinguish them both: for, if the sun moves to one of them, it may be called the apex of its motion, and as the stars will then have a parallactic motion to the opposite one, the appellation of a parallactic center may very properly be given to it. The latter falling into the southern hemisphere, among constellations not visible to us, I shall only mention their opposite intersections; and of these I find no less than ten that are made by stars of the first magnitude, in a very limited part of the heavens, about the constellation of Hercules. Upon all the remaining surface of the same globe there is not the least appearance of any other than a promiscuous situation of intersections; and of these only a single one is made by arches of principal stars.

The ten intersecting points made by the brightest stars are as follows. The 1st is by Sirius and Arcturus, in the mouth of the Dragon. The 2d by Sirius and Capella, near the following hand of Hercules. The 3d by Sirius and Lyra, between the hand and knee of Hercules. The 4th by Sirius and Aldebaran, in the following leg of Hercules. The 5th by Arcturus and Capella, north of the preceding wing of the Swan. The 6th by Arcturus and Aldebaran, in the neck of the Dragon. The 7th by Arcturus and Procyon, in the preceding foot of Hercules.

The 8th by Capella and Procyon, south of the following hand of Hercules. The 9th by Lyra and Procyon, preceding the following shoulder of Hercules. And the 10th is made by Aldebaran and Procyon, in the breast of Hercules.

The following Table gives the calculated situation of these ten intersections in right ascension and north polar distance.

Table I.

No.	Right Ascension.			Polar Distance.		
1	255°	39′	50″	36°	41′	34″
2	275	9	32	64	21	48
3	272	23	58	58	23	24
4	263	25	38	44	39	47
5	290	0	58	32	7	23
6	267	2	19	33	57	20
7	235	3	13	46	21	34
8	272	51	49	73	7	56
9	266	46	49	66	48	11
10	260	1	29	60	59	34

We might rest satisfied with having shown that the parallactic effect of which we are in search is plainly to be perceived in the motion of the brightest stars; however, by way of further confirmation, we may take in some large stars of the next order, in whose motions evident marks of the influence of parallax may likewise be perceived. When the intersections made by their proper motions and the arches in which the stars of the first magnitude are moving, are examined, we find no less than fifteen which unite with the former ten, in pointing out the same part of the heavens as a parallactic center. It will be sufficient only to mention the opposite

points of the situation of these intersections, and the stars by which they are made, without giving a calculated table of them.

The 1st is the following leg of Hercules, and is made by Sirius and β Tauri. The 2d is also in the following leg of Hercules, by Sirius and α Andromedæ. The 3d is in the following hand of Hercules, by Sirius and α Arietis. The 4th in the neck of the Dragon, by Arcturus and β Tauri. The 5th between the Lyre and the northern wing of the Swan, by Capella and α Andromedæ. The 6th near the following hand of Hercules, by Capella and α Arietis. The 7th preceding the head of Hercules, by Lyra and β Tauri. The 8th between the Lyre and northern wing of the Swan, by Lyra and α Andromedæ. The 9th in the following arm of Hercules, by Lyra and α Arietis. The 10th in the following leg of Hercules, by Aldebaran and β Tauri. The 11th in the following leg of Hercules, by Aldebaran and α Andromedæ. The 12th in the head of Hercules, by Aldebaran and α Arietis. The 13th in the following arm of Hercules, by Procyon and β Tauri. The 14th in the back of Hercules, by Procyon and α Andromedæ. And the 15th near the following arm of Hercules, is made by Procyon and α Arietis.

An argument like this, founded upon the most authentic observations, and supported by the strictest calculations, can hardly fail of being convincing. And though only the ten principal apices of the twenty-five that are given have been calculated, the other fifteen may nevertheless be depended upon as true to less than one degree of the sphere.

Changes in the Position of double Stars.

We have lately seen that the alterations in the relative situation of a great number of double stars may be accounted for by a parallactic motion. Among the 56 stars which I have given, the changes of more than half of them appear to be of this nature; and it will certainly be more eligible to ascribe them to the effect of parallax than to admit so many separate motions in the different stars; especially when it is considered that if the alterations of the angle of position were owing to a motion of the largest star of each set, the direction of such motions must, in contradiction to all probability, tend nearly to one particular part of the heavens.

This argument, drawn from the change of the position of double stars, may be considered as deriving its validity from the same source with the former, namely, the parallactic motions of at least 28 more stars, pointing out the same apex of a solar motion by their direction to its opposite parallactic center.

Incongruity of proper Motions.

It may be remarked that the proper motions of the stars, if they were in reality such as they appear to be, would contain a certain incongruous mixture of great velocity and extreme slowness. Arcturus alone describes annually an arch of more than two seconds: Aldebaran hardly one-tenth and a quarter of a second: Rigel little more than one-tenth and a half: even Lyra moves barely three and a quarter tenths of a second, while Procyon has almost four times that velocity. Out of 36 stars whose proper motion we have examined, there are 15 that do not reach two-tenths of a second: β Virginis moves

seventy-seven hundredths, and α Cygni only six. But it will be shown, when the direction and velocity of the solar motion come to be explained, that these kind of incongruities are mere parallactic appearances; and that there is so general a consistency among the real motions of the stars, that Arcturus is in no respect singled out as a star whose motion is far beyond the rest.

By giving this remark a place among the reasons for admitting a solar motion, it is not intended to lay any particular stress upon it; for it may be objected that our idea of the congruence or harmony of the celestial motions can be no criterion of their real fitness and symmetry. But when such discordant proper motions as those I have mentioned in stars of no very different lustre are under consideration, and may be easily shown to be only parallactic phenomena, the method by which this can be done must certainly appear eligible, and when added to many other inducements, will throw some share of weight into the scale.

Sidereal Occultation of a small Star.

Of nearly the same importance with the former argument is the account of the occultation of a small star by a large one, which I have given in my last Paper. When the solar motion has been established, we shall prove that the vanishing of the small star near δ Cygni, as far as we can judge at present, is only a parallactic disappearance. It must be granted that a real motion of the large star would also explain the same phenomenon; but then again, this star must be supposed to move towards the very same parallactic center which the changes in the

position of other double stars point out, and this cannot be probable.

Direction of the solar Motion.

From what has been said, I believe the expedience of admitting a solar motion will not be called in question; our next endeavour therefore must be to investigate its direction.

To return to the before mentioned intersections of the arches, in which the proper motions of the stars are performed, I shall begin by proving that when the proper motions of two stars are given, an apex may be found, to which, if the sun be supposed to move with a certain velocity, the two given motions may then be resolved into apparent changes, arising from sidereal parallax, the stars remaining perfectly at rest.

Let the stars be Arcturus and Sirius, and their annual proper motions as given in the Astronomer Royal's Tables.

When the annual proper motion of Arcturus, which is $-1'',26$ in right ascension, and $+1'',72$ in north polar distance, is reduced by a composition of motions to a single one, it will be in a direction which makes an angle of $55°\ 29'\ 42''$ south-preceding with the parallel of Arcturus, and of a velocity so as to describe annually $2'',08718$ of a great circle.

The annual proper motion of Sirius, $-0'',42$ in right ascension, and $+1'',04$ in north polar distance, by the same method of composition, becomes a motion of $1'',11528$, in a direction which makes an angle of $68°\ 49'\ 41''$ south-preceding with the parallel of Sirius.

By calculation, the arches in which these two stars move, when continued, will meet in what I have called their parallactic center, whose right ascension is $75°\ 39'\ 50''$, and south polar

distance is 36° 41′ 34″. The opposite of this, or right ascension 255° 39′ 50″, and north polar distance 36° 41′ 34″, is what we are to assume for the required apex of the solar motion.

When a star is situated at a certain distance from the sun, which we shall call 1; and 90° from the apex of the solar motion, its parallactic motion will be a maximum. Let us now suppose the velocity of the sun to be such that its motion, to a person situated on this star, would appear to describe annually an arch of 2″,84825, or, which is the same thing, that the star would appear to us, from the effect of parallax, to move over the above mentioned arch in the same time.

To apply this to Arcturus, we find by calculation that its distance from the apex of the solar motion is 47° 7′ 6″; its parallactic motion therefore, which is as the sine of that distance, will be 2″,08718; and this, as has been shown, is the apparent motion which observation has established as the proper motion of Arcturus.

In the next place, if we admit Sirius to be a very large star situated at the distance 1,6809 from us, and compute its elongation from the apex of the solar motion, we shall find it 138° 50′ 14″,5. With these two data we calculate that its parallactic motion will be $\frac{\varphi \cdot S}{r \cdot d} = sp =$ 1″,11528; and this also agrees with the apparent motion which has been ascertained by observation as the proper motion of Sirius.

Now since, according to the rules of philosophising, we ought not to admit more motions than will account for the observed changes in the situation of the stars, it would be wrong to have recourse to the motions of Arcturus and Sirius, when that of the sun alone will account for them both; and this consideration would be a sufficient inducement for us to

fix at once on the calculated apex, as well as on the relative distances that have been assigned to these stars, if other proper motions could with equal facility be resolved into similar parallactic appearances. But from the nature of proper motions, it follows, that when a third star does not lead us to the same apex as the other two, its apparent motion cannot be resolved by the effect of parallax alone. And to enhance our difficulties, the number of apices, that would be required to solve all proper motions into parallactic ones, increases not as the number of stars admitted to have proper motions, but, when their situation happens to be favourable, as the sum of an arithmetical series of natural numbers, beginning at 0, continued to as many terms as there are stars admitted: so that if two stars give only one apex, one star added to it will give three apices; and ten, for instance, will give no less than 45, and so on.

The method of reasoning which, on this subject, I have adopted, is so closely connected with astronomical observations that I shall keep them constantly in view; and therefore shall illustrate what has been advanced, by taking in Capella as a third star. The three apices which then are pointed out will be that in the mouth of the Dragon, by Arcturus and Sirius; a second under the northern wing of Cygnus, by Arcturus and Capella; and a third in the following hand of Hercules, by Sirius and Capella. The calculation of them is in Table I.

The annual proper motions of our third star in Dr. MASKELYNE's Tables are $+ 0'',21$ in right ascension, and $+ 0'',44$ in north polar distance; and by calculation these quantities give an annual motion of $0'',46374$ to Capella, in a direction which makes an angle of $71° \ 35' \ 22'',4$ south-following with the parallel of this star.

The distance of Capella from the same calculated apex of the solar motion, by which we have already explained the apparent motions of the other two stars, is 80° 54′ 46″; and, admitting again the velocity of the sun towards the same point as stated before, it will occasion a parallactic motion of Capella, in a direction 89° 54′ 48″ south-following its parallel, amounting to 2″,8125. In this calculation Capella has been taken for a star of the first magnitude, supposing its distance from us to be equal to that of Arcturus.

By constructing then a triangle, the three sides of which will represent the three motions which every star must have that is not at rest in space; we have one of the sides, representing the apparent motion of the star, equal to 0″,4637; the other side, being the parallactic motion of the star 2″,8125; and the included angle 18° 19′ 27″. From these data we obtain the third side, representing the real motion of the star, which will be 2″,3757. By the given situation of this triangle with respect to the parallel of declination of Capella, the angle of the real motion will also be had, which is 86° 34′ 11″ north-following the parallel of this star. A composition of the parallactic and the real motion in the directions we have assigned, will produce the annual apparent motion which has been established by observation.

But to apply what has been said to our present purpose, it may be observed, that although we have accounted for the proper motion of our third star by retaining the same apex of the solar motion, which has given us an explanation of the apparent motions of the other two, yet in doing this we have been obliged to assign a great degree of real motion to Capella; and to this it may be objected, that we can have no authority

to deprive Arcturus and Sirius of real motions, in order to give one of the same nature to our third star: and indeed to every star that has a proper motion which does not tend to the same parallactic center as the motions of Arcturus and Sirius.

This objection is perfectly well founded, and I have given the above calculation on purpose to show that, when we are in search of an apex for the solar motion, it ought to be so fixed upon as to be equally favourable to every star which is proper for directing our choice. Hence a problem will arise, in our present case, how to find a point whose situation among three given apices shall be so that, if the sun's motion be directed towards it, there may be taken away the greatest quantity of proper motion possible from the given three stars. The intricacy of the problem is greater than at first it may appear, because by a change of the distance of the apex from any one of the stars, its parallactic motion, which is as the sine of that distance, will be affected; so that it is not the mere alteration of the angle of direction, which is concerned. However, it will not be necessary to enter into a solution of the problem; for it must be very evident that a much more complex one would immediately succeed it, since three stars would certainly not be sufficient to direct us in our present endeavour to find the best situation of an apex for the solar motion; I shall therefore now leave these stars, and the apices pointed out by them, in order to proceed to a more general view of the subject.

We have already seen that the brightest stars are most proper for showing the effect of parallax, and that in our search after the direction of the solar motion, our aim must be to reduce the proper motions of the stars to their lowest

quantities. The six principal stars, whose intersecting arches have been given, when their proper motions in right ascension and polar distance are brought into one direction, will have the apparent motions contained in the following Table.

Table II.

Names of the Stars.	Direction of the apparent Motions.	Quantities of the apparent Motions.
Sirius -	68° 49′ 40″,7 south-preceding	1″,11528 per year
Arcturus -	55 29 42,0 south-preceding	2,08718 ———
Capella -	71 35 22,4 south-following	0,46374 ———
Lyra - -	56 20 57,3 north-following	0,32435 ———
Aldebaran	76 29 37,3 south-following	0,12341 ———
Procyon -	50 2 24,5 south-preceding	1,23941 ———
	Sum of the apparent motions	5″,35337

We must now recur to what has been said, when the construction of the triangle expressing the three motions of a star, that is not at rest, was explained; and, as we are to find out a solar motion which will require the least real motion in our six stars, an attention to this triangle will be of considerable use; for when the line *pa*, Fig. 1, which represents the real motion, is brought into the situation *ma*, where it is perpendicular to *sp*, the real motion which is required will then be a minimum. It also follows, from the construction of the same triangle, that if by the choice of an apex for the solar motion we can lessen the angle made at *s* by the lines *sp* and *sa*, we shall lessen the quantity of real motion required to bring the star from the parallactic line *spm* to the observed position *a*.

It has already been shown, in the case of Sirius and Arcturus,

that when two stars only are given, the line sp may be made to coincide with the lines sa, of both the stars, whereby their real motions will be reduced to nothing. It has also been proved, by adding Capella to the former two, that when three stars are concerned, some real motion must be admitted in one of them. Now, since all parallactic motions are directed to the same center, a single line may represent the direction of the effect of the parallax, not only of these three stars but of every star in the heavens. According to this theory, let the line sP or sS, in Fig. 2, stand for the direction of the parallactic motion of the stars; and as in the foregoing Table we have the angles of the apparent motion of six stars with the parallel of each star, we must now also compute the direction of the line sP or sS with the parallels of the same stars. This may be done as soon as an apex for the solar motion is fixed upon. The difference between these angles and the former will give the several parallactic angles Psa or Ssa, required for an investigation of the least quantity ma, belonging to every star.

For instance, let the point towards which we may suppose the sun to move, be λ Herculis; and calculating the required angles of the direction in which the effect of parallax will be exerted, with the six stars we have selected for the purpose of our investigation, we find them as in the following Table.

Table III.

Angles of the parallactic Motion with the Parallel.

Sirius - - -	32°	54′	8″,5	south-preceding.
Arcturus - - -	17	23	45,7	south-preceding.
Capella - - -	85	10	3,9	south-following.
Lyra - - - -	35	59	49,5	north-following.
Aldebaran - -	71	21	35,4	south-following.
Procyon - -	47	43	44,6	south-preceding.

The difference between these parallactic, and the former apparent angles, with the parallel of each star, will give the required angles for our second figure. They will be as follows.

Table IV.

Angles of the apparent with the parallactic Motion.

Sirius - - -	35°	55′	32″,2	south-following.
Arcturus - - -	38	5	56,3	south-following.
Capella - - -	13	34	41,5	south-following.
Lyra - - - -	20	21	7,8	north-preceding.
Aldebaran - - -	5	8	1,9	south-preceding.
Procyon - - -	2	18	39,9	south-following.

By these angles, with the assistance of the lines *sa*, whose lengths represent the annual quantity of the apparent motions as given in our former Table, the Figure No. 2 has been constructed. When the situation of these angles is regulated as in that figure, we may draw the several lines *ma* perpendicular to SP, and, by computation, their value and sum will be obtained as follows.

Table V.

Quantities and Sum of the least real Motions.

Sirius	- - -	0",65437
Arcturus	- - -	1,28784
Capella	- - -	0,10887
Lyra	- - -	0,11281
Aldebaran	- - -	0,01104
Procyon	- - -	0,04998
	Sum	2",22491.

The result of this investigation is, that by admitting a motion of the sun towards λ Herculis, the annual proper motions of our six stars, of which the sum is 5",3537, may be reduced to real motions of no more than 2",2249.

When first I proposed λ Herculis as an apex for the solar motion, it may be remembered that a reference to future observations was made for obtaining greater accuracy.* Such observations we have now before us, in the valuable Tables from which I have taken the proper motions of the six stars; and I shall prove that, with their assistance, we may fix on a solar motion that will be considerably more favourable.

We have already shown, that to ascertain the precise place of the best apex is attended with some difficulty; but from the inspection of the figure which represents the quantities of real motion required when λ Herculis is fixed upon, it will be seen that, by a regular method of approximation, we may turn the line SP into a situation where all the angles of the

* See Phil. Trans. for 1783, p. 273, line 8; and page 274, line 4.

apparent motion of the six stars, will be much reduced. The quantities which are required for constructing another figure to represent the threefold motions of our six stars, when a different apex is fixed upon, are to be found by the same method we have pursued in the instance of λ Herculis; and the figure that has been given with respect to that star, shows evidently that the parallactic line SP should be turned more towards the line *sa*, representing the apparent motion of Sirius. We shall accordingly try a point near the following knee of Hercules, whose right ascension is 270° 15′, and north polar distance 54° 45′.

The result of a calculation of the angles and the least quantities of real motion of our six stars, according to this apex, is collected in the following Table, and represented in Fig. 3.

Table VI.

Stars.	Angles of the parallactic Motion with the Parallel.	Angles of the apparent with the parallactic Motion.	Least Quantities of the real Motion.
	° ′ ″	° ′ ″	″
Sirius -	68 51 5 south-preceding	0 1 25 south preceding	0,0004561
Arcturus	29 30 32 south-preceding	25 59 10 south-following	0,9145072
Capella -	77 54 0 south-following	6 18 38 south-following	0,0509727
Lyra -	27 38 47 north-following	28 42 9 north-preceding	0,1557761
Aldebaran	66 20 17 south-following	10 9 21 south preceding	0,0217607
Procyon -	64 48 27 south-following	14 46 1 south-preceding	0,3159051
		Sum	1″,4593779

By this Table it appears that the annual proper motion of our six stars may be reduced to 1″,4594, which is 0″,7655 less than the sum in the 5th Table, where the apex was λ Herculis.

In the approximation to this point it appeared, that when the line of the parallactic motion of Sirius is made to coincide

with its apparent motion, we may soon obtain a certain minimum of the other parallactic motions; but as Sirius is not the star which has the greatest proper motion, it occurred to me that another minimum, obtained from the line in which Arcturus appears to move would be more accurate; for, on account of its great proper motion, we have reason to suppose it more affected than other stars, by the parallax arising from the motion of the sun; and, with a view to this, I soon was led to a point not only in the line of the apparent motion of Arcturus, but equally favourable to Sirius and Procyon, the remaining two stars that have the greatest motions.

If the principle of determining the direction of the solar motion by the stars which have the greatest proper motion be admitted, the following apex must be extremely near the truth; for, an alteration of a few minutes in right ascension or polar distance either way, will immediately increase the required real motion of our stars. Its place is: right ascension 245° 52′ 30″, and north polar distance 40° 22′.

The calculated motions of the same stars by this apex will be as in the following Table, and are delineated in Fig. 4.

Table VII.

Stars.	Angles of the parallactic Motion with the Parallel.	Angles of the apparent with the parallactic Motion.	Least Quantities of the real Motion.
	° ′ ″	° ′ ″	″
Sirius -	58 24 56 south-preceding	10 24 44 following -	0,20157
Arcturus	55 29 45 south-preceding	0 0 3 preceding -	0,00003
Capella -	83 44 17 south-preceding	24 40 21 following -	0,19358
Lyra -	36 28 33 south-following	92 49 30 following -	0,32396
Aldebaran	89 48 35 south preceding	13 18 58 following -	0,02842
Procyon -	59 43 10 south-preceding	9 40 46 preceding -	0,20839
		Sum	0″,95595

The sum of the real motions required, with the apex of the solar motion above mentioned, is less in this Table than that in the former by 0",50343.

In these calculations we have proceeded upon the principle of obtaining the least possible quantity of real motion, by way of coming at the most favourable situation of a solar apex, and have proved that the sum of the observed proper motions of the six principal stars, amounting to 5",3534, may be the result of a composition of two other motions, and that the real motions of these stars, if they could be reduced to their smallest possible quantities, would not exceed 0",9559.

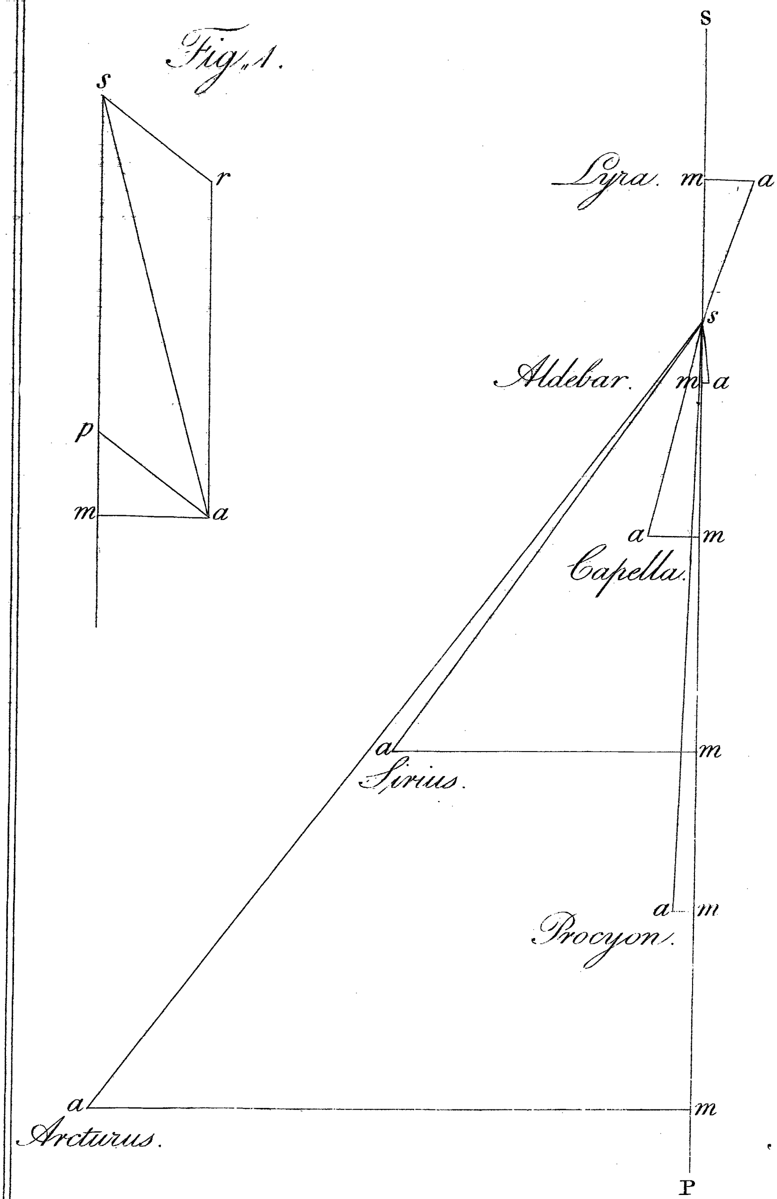
But as I do not intend to assert that these real motions can be actually brought down to the low quantities that have been mentioned, it will be necessary to show that the validity of the arguments for establishing the method I have pursued will not be affected by that circumstance. In the first place then, we should consider that although the great proper motions of Arcturus, Procyon, and Sirius, are strong indications of their being affected by parallax, it does not follow, nor is it probable, that the apparent changes of the situation of these stars should be intirely owing to solar motion; on the contrary, we may reasonably expect that their own real motions will have a great share in them. Next to this, it is evident that in the case of parallactic motions the distance of a star from the sun is of material consequence; and as this cannot be assumed at pleasure, we are consequently not at liberty to make the parallactic motion *sp* in Fig. 1, equal to the line *sm* of the same figure; hence it follows, that the real motion of the star cannot be from *m* to *a*, as the foregoing calculations have supposed; but will be from *p* to *a*. It is however very evident, that if *ma* be

a minimum, the line *pa*, when *sp* is given, will also be a minimum; and if all the *ma*'s in Fig. 4 are minima, it follows also that all the *sp*'s, whatever they may be, will give the *pa*'s as small as possible: and this is the point that was to be established.

Whatever therefore may be the sum of real motions required to account for the phenomena of proper motions, our foregoing arguments cannot be affected by the result; for, as by observation it is known that proper motions do exist, and since no solar motion can resolve them intirely into parallactic ones, we ought to give the preference to that direction of the motion of the sun which will take away more real motion than any other, and this, as we have shown, will be done when the right ascension of the apex is 245° 52′ 30″, and its north polar distance 40° 22′.

Fig. 1.

Fig. 2.


s
r
p
m
a
S
Lyra. m a
s
Aldebar. m a
a m
Capella.
a m
Sirius.
a m
Procyon.
a m
Arcturus.
P

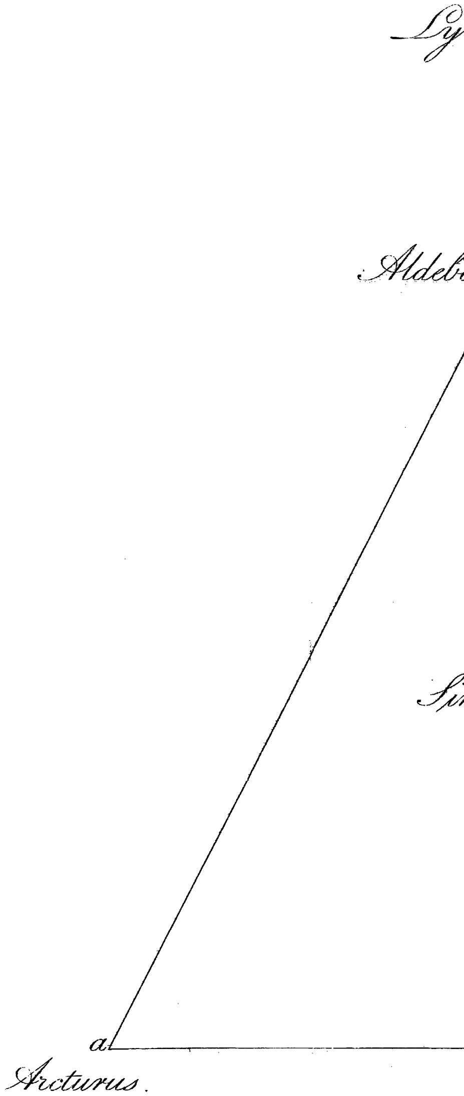

a
Arcturus.

Fig. 3.

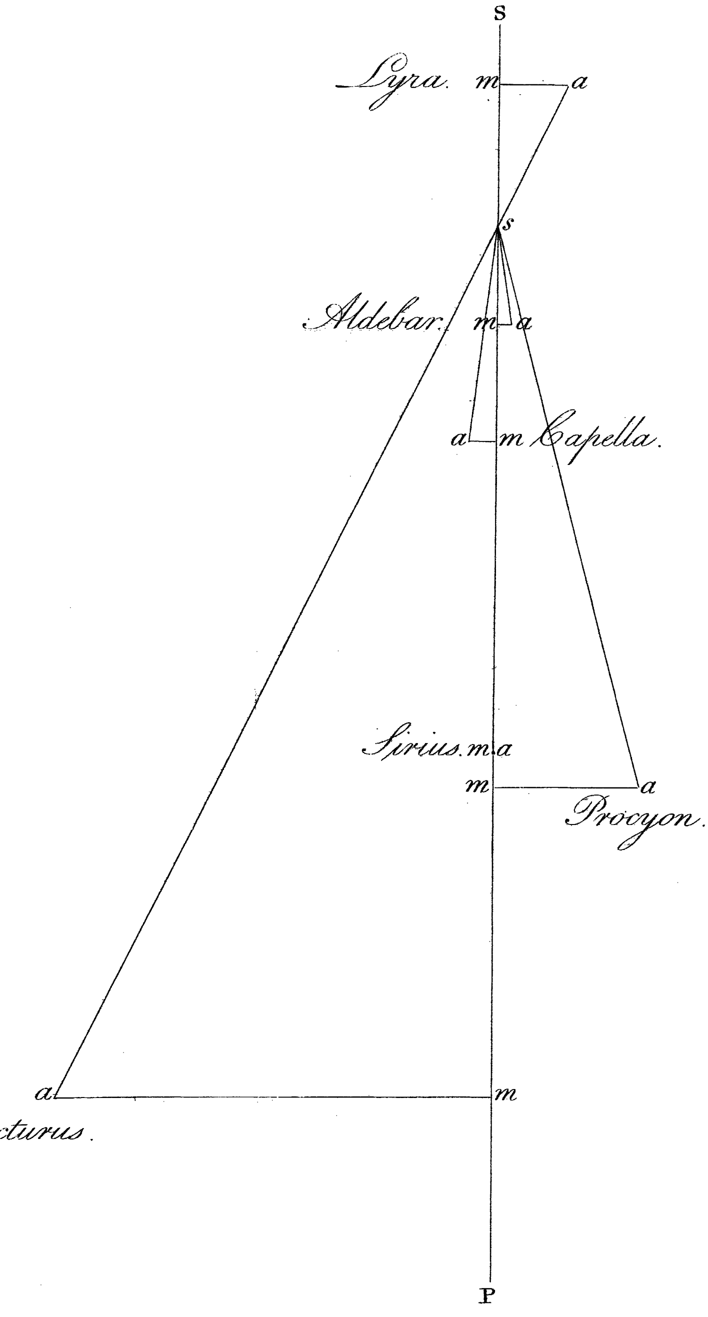
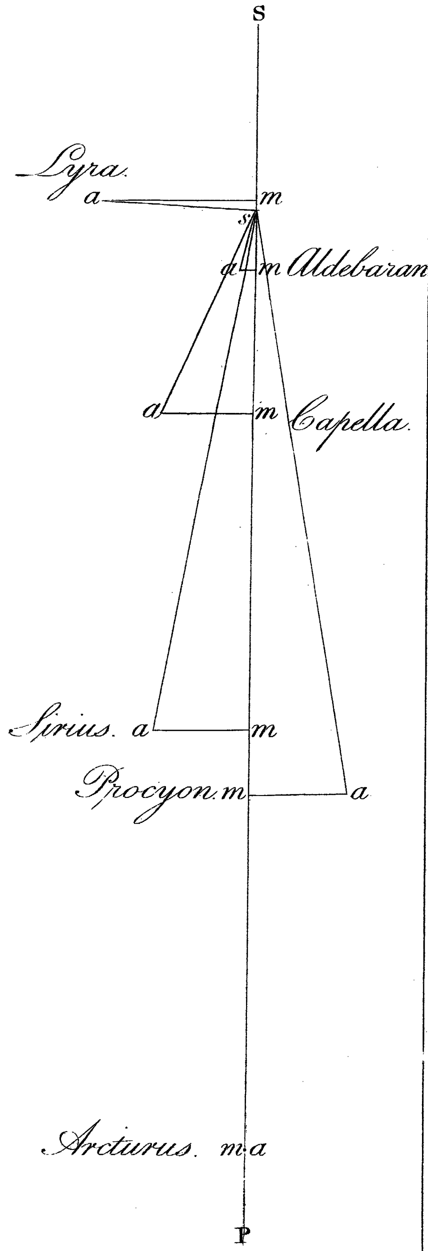


Fig. 4.



Basire sc.